Interaction of Anti-Lock Braking Systems with Tire Torsional Dynamics

John Adcox*
jadcox@clemson.edu
Clemson University – International Center for Automotive Research
4 Research Dr., CGEC, Greenville, SC, 29607, USA

Dr. Beshah Ayalew
beshah@clemson.edu
Clemson University – International Center for Automotive Research
4 Research Dr., Room 342 CGEC, Greenville, SC, 29607, USA

Dr. Tim Rhyne
tim.rhyne@us.michelin.com
Michelin America Research Corporation
515 Michelin Rd, Greenville, SC 29605, USA

Steve Cron
steve.cron@us.michelin.com
Michelin America Research Corporation
515 Michelin Rd, Greenville, SC 29605, USA

Mike Knauff
mike.knauff@us.michelin.com
Michelin America Research Corporation
515 Michelin Rd, Greenville, SC 29605, USA

Presented at the
September 2011
Meeting of the Tire Society

*Presenter / Corresponding Author
Interaction of Anti-Lock Braking Systems with Tire Torsional Dynamics


ABSTRACT: A tire’s torsional dynamics couple the responses of wheel/hub inertia to that of the ring/belt inertia. Depending on the effective stiffness, damping and mass distribution of the tire, the ensuing deflections between the wheel and the ring can cause significant errors in the estimation of the tire’s longitudinal slip from wheel speed measurements. However, this remains the established approach for constructing anti-lock braking (ABS) control algorithms. Under aggressive braking events, the errors introduced by torsional dynamics may significantly affect the ABS algorithm and result in less than optimal braking performance.

This paper investigates the interaction of tire-torsional dynamics and ABS control using a comprehensive system model that incorporates sidewall flexibility, transient and hysteretic tread-ground friction effects, and the dominant dynamics of a hydraulic braking system. It considers a wheel/hub acceleration-based ABS controller that mimics the working steps of a commercial ABS algorithm. Results from multiple sensitivity studies show a strong correlation of stopping distances and ABS control activity with design parameters governing tire/wheel torsional response, and the filter cutoff frequency of the wheel acceleration signals used by the controller.

KEY WORDS: Anti-locking Braking systems, tire torsional dynamics, wheel acceleration control, LuGre tread-friction model

Introduction

ABS control algorithms have been commercially available on vehicles for years and have been researched for even longer. Most of the published research on ABS algorithms is based on simplified rigid wheel dynamics models where the primary focus is on accommodating tire-ground slip conditions on various road surfaces. With advances in tire/wheel technology, including available aftermarket choices, the different torsional dynamic properties of the tire/wheel systems cannot be sufficiently captured using rigid wheel assumptions for the entire tire/wheel system. For example, there are drastic differences in torsional dynamic properties between an inflated or deflated run-flat tire. ABS algorithms that embed rigid wheel assumptions are likely to give compromised performance in terms of achievable stopping distances when working with such drastically different tire/wheel designs. However, there remains very little published research that looks into how these design variations interact with and influence the performance of ABS controllers.
Various tire models have been developed in order to better approximate the transient dynamics of a tire [1][2][3][4]. And multiple authors [5][6][7] have modeled and simulated ABS control structures that are combined with these various flexible tire models in order to see their effect on braking performance. These works did not put emphasis on the analysis of the robustness of ABS controller’s achievable stopping distance to changes in the tire/wheel parameters. However, there has been some anecdotal experimental evidence that suggests ABS interacts differently with different tire designs during hard braking events. For example, testing performed at Michelin showed that, on a wet asphalt surface, a tire that has a low torsional stiffness performed approximately 30% worse than a tire with a standard torsional stiffness when fitted to the same vehicle with ABS. However, when the ABS was deactivated the stopping distance for the two tires were almost equal.

In light of the above, the objective of this work is to study the interactions of tire/wheel designs with the workings of a typical commercial ABS control system. To this end, a detailed simulation model of the braking system dynamics of a small passenger vehicle is developed. It includes tire sidewall torsional deflection, dynamic tread-ground friction effects, and brake hydraulics. This integrated model is used to conduct sensitivity studies on the achievable braking performance by changing the parameters of the ABS control algorithm and the various tire and wheel design parameters, namely, sidewall flexibility and damping, and inertia distributions.

This paper is organized as follows. First, the system model adopted for the ensuing analysis is briefly described. Then, the open-loop (i.e. without an ABS algorithm) response of the coupled system is analyzed. This is followed by a brief description of a wheel-acceleration based ABS controller. The closed-loop system is then used to perform a series of sensitivity studies considering changes in the tire’s torsional characteristics and controller settings. The conclusions drawn from these sensitivity studies are summarized at the end of the paper.

System Modeling

Tire Model

The tire/wheel model that is used throughout this paper only includes the torsional deflection of the sidewall, as this is considered to be the dominant effect on the response of the tire/wheel system onto which the braking inputs are applied. As such, a two-inertia model representing the
ring and hub is considered as shown in Figure 1. The sidewall’s torsional stiffness and damping coefficient are denoted by $K_T$ and $C_T$, respectively. A tangential tread-deflection model is also incorporated using the Average Lumped Parameter LuGre friction model detailed in [8] and also discussed in [9][10][11][12]. The schematic for this model is shown in Figure 2, where $K_{tread}$ and $C_{tread}$ are stiffness and damping coefficients appearing in the LuGre model [16].

Figure 1: Hub/Tire Model  Figure 2: Schematic for the LuGre Friction Model

Considering a quarter vehicle model along with the above tire/wheel and tread/ground friction model, the equations describing the system reduce to the following:

\[
J_w \frac{d\omega_w}{dt} = K_t(\theta_r - \theta_w) + C_t(\omega_r - \omega_w) - T_b \tag{1}
\]

\[
J_r \frac{d\omega_r}{dt} = F_t R_r - K_t(\theta_r - \theta_w) - C_t(\omega_r - \omega_w) \tag{2}
\]

\[
\frac{m_v}{4} \frac{dv}{dt} = -F_t \tag{3}
\]

\[
F_t = F_z(K_{tread}z + C_{tread}\dot{z}) \tag{4}
\]

\[
V_r = V - R_r \omega_r \tag{5}
\]

\[
\dot{z} = V_r - \frac{K_{tread}|V_r|}{g(V_r)} z - k|\omega_r| R_r z \tag{6}
\]

\[
g(V_r) = \mu_c + (\mu_s - \mu_c)e^{-|V_r/V_s|^a} \tag{7}
\]

Here, Equation 1 and Equation 2 represent the hub and ring dynamics, respectively. $J_w$ and $J_r$ designate the hub/wheel and ring inertias, $K_t$ designates the sidewall stiffness, $C_t$ designates the
damping coefficient of the sidewall, $T_b$ designates the braking torque, and $F_t$ designates the ground force. Equation 3 gives the longitudinal braking dynamics of the quarter vehicle, where aerodynamic and rolling resistance contributions have been neglected. In this work, vehicle parameters for a small passenger vehicle (1991 Mazda Miata) are considered.

Equations 4-7 represent the equations for the Average Lumped Parameter LuGre friction model [8][9][10][11][12]. In Equation 4, the ground force ($F_t$) equals the sum of effective tread stiffness and tread damping forces times the normal force ($F_z$) on the tire, where $z$ represents the effective or average tread/bristle deflection in the contact zone. Equation 5 shows the relationship between the relative sliding velocity ($V_r$), and the vehicle velocity ($V$), and the tangential velocity of the ring. Equation 6 represents the tread deflection rate as a function of the relative sliding velocity, the tread stiffness, a load distribution factor for the contact patch ($k$) and the sliding/Strubeck friction function $g(Vr)$. Equation 7 defines this Strubeck friction function for the friction coefficient of the tread/ground interface as a function of the sliding velocity. In this equation, $\mu_s$ and $\mu_c$ represent the static and coulomb friction coefficients, respectively, $V_s$ is the Strubeck sliding velocity, and $\alpha$ is a shaping factor that is used to capture the shape of the friction-slip curve. For the investigations in this work, the ‘Strubeck’ friction curve has been extrapolated from experimental data for a wet surface where the static ($\mu_s$) and kinetic ($\mu_c$) coefficients of friction are 0.75 and 0.4, respectively, and the shaping factor ($\alpha$) has been determined as 0.75.

**Basic Responses of Tire Model**

It is useful to quantify certain linearized characteristics, such as the natural frequency and damping ratio, by analyzing the tire/wheel system in a free-free state where the nonlinear friction forces and the tread dynamics are ignored. These are given as follows [18]:

$$\omega_n = \frac{\sqrt{K_T(0r+J_w)}}{1r+J_w}$$  \hspace{1cm} (8)

$$\zeta = \frac{C_T}{2} * \frac{1r+J_w}{K_T(0r+J_w)} \hspace{1cm} (9)$$

Table 1 below shows computed values of these parameters for two tire designs for the same vehicle. The torsional stiffness values for the two designs are experimentally determined, and the torsional damping coefficient was merely selected to keep the damping ratio at the typical value.
of 0.05. Tire 1 is a low torsional stiffness tire (lower torsional natural frequency), and Tire 2 is a tire with the torsional stiffness of a stock pneumatic tire.

<table>
<thead>
<tr>
<th></th>
<th>$K_T \ [N \cdot m \cdot rad.]$</th>
<th>$C_T \ [N \cdot m \cdot sec. \cdot rad.]$</th>
<th>$J_r \ [kg \cdot m^2]$</th>
<th>$J_w \ [kg \cdot m^2]$</th>
<th>$\omega_n \ [Hz]$</th>
<th>$\zeta$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Tire 1</strong></td>
<td>7616</td>
<td>2.5</td>
<td>1</td>
<td>0.093</td>
<td>47.6</td>
<td>0.05</td>
</tr>
<tr>
<td><strong>Tire 2</strong></td>
<td>19438</td>
<td>4</td>
<td>1</td>
<td>0.093</td>
<td>76.1</td>
<td>0.05</td>
</tr>
</tbody>
</table>

**Brake Hydraulics Model**

Figure 3 shows the main components of such a hydraulic braking system configured for ABS [13]. In the model adopted, the hydraulic dynamics before the inlet and outlet valves have been ignored, assuming that the valve responses and pressure (compressibility effects) dynamics dominate the hydraulic dynamics. This is equivalent to assuming that the build-up phase for the master cylinder pressure is neglected and that the oil pump is ideal. Equations 10-14 list the equations derived for describing the dynamics of the brake system under these assumptions.

**Figure 3: Schematic of Brake Hydraulics [13]**

1. Oil Reservoir Tank  
2. Master Cylinder   
3. Oil Pump           
4. Check Valve       
5. Inlet Valve       
6. Outlet Valve      
7. Disk Brake        
8. Wheel/Tire Assembly
Equation 10 represents the caliper cylinder pressure dynamics as a function of the bulk modulus and volume of the fluid and the flow rate through the brake lines. Equation 11 and Equation 12 represent the flow rates through the inlet and outlet valves, respectively. Equation 13 models how the effective valve area changes with the valve input, the gain of the valve, delays and its time constant. And lastly, Equation 14 converts the pressure from the brake lines into the torque that is applied by the caliper on the wheel hub given two brake pads at a given radius from the wheel center at a given pad friction coefficient. A more detailed model of the hydraulic brake system is given in reference [13].

The brake system parameters have been determined from both physical measurements and a reference [17] to represent typical characteristics of a hydraulic braking system for a small passenger car (the 1991 Mazda Miata). The following values were used throughout the simulations: $\beta = 1$ GPa; $\rho = 0.85$ kg/L; $P_m = 5$ MPa; $A_{max} = 0.5$ mm$^2$; $C_d = 0.6$; $V = 50$ cm$^3$.

The time constant for the caliper pressure dynamics is found to be of the order of 15 ms. When cascaded with the valve dynamics, which has a time constant of 10 ms, this produces an overall brake caliper pressure response (to valve input) on the order of 20 ms.

**Open-Loop Response of Tire and Brake System**

The tire and brake system models presented above are connected together and the responses of the combined system to step changes in valve voltage (input, and output valves) are analyzed. Figure 4 shows this transient response in terms of the longitudinal
force coefficient ($\mu$) vs. the wheel and ring slip ratios ($l_w$, $l_r$) at each instant during this simulated hard braking event. These quantities are defined as:

$$l_w = \frac{F_t}{F_z} = 1 - \frac{wR}{V} \quad \text{and} \quad l_r = 1 - \frac{R}{V}$$

It can be seen that for the low torsional stiffness tire (Tire 1), the force coefficient builds up to the Stribeck curve, at approximately 10% ring slip ratio, and then smoothly follows just under the Stribeck friction curve until it reaches full lockup. It also shows that for a given value of the force coefficient, the wheel slip ratio lags the ring slip ratio during the force build up phase and once it reaches the peak force coefficient, the wheel slip ratio exhibits oscillations around the ring slip ratio. We can make parallel observations on the effect of sidewall flexibility by referring to the right side of Figure 4, which shows the torsional angle between the wheel and the ring during the same hard braking event. During initial force build up, there is an increase in the relative torsional angle until a peak value of $\mu$ is achieved. Then the wheel and ring oscillate relative to each other with an average twist of around 0.029 radians until the wheel locks-up and the ring continues to oscillate dramatically about the wheel.

**Figure 4: Open-Loop Response with Torsional Dynamics**

These results confirm that, in the presence of tire torsional flexibility, there is a distinction between the behavior of the ring and wheel slip ratios during a hard braking event and one
should therefore expect some interaction with an ABS controls system that attempts to influence this very dynamics.

**Acceleration-based ABS Controller**

For this work, an acceleration-based ABS controller has been modeled after the ABS control algorithm outlined by the ABS system supplier Bosch [14]. The ABS controller cycles through various control phases is designed around a set of predetermined thresholds that are highlighted in Figure 5. While a brief description of the cycles and thresholds is given below, the reader is referred to sources [14][15] for details.

![Bosch Wheel-Acceleration Based ABS Algorithm](image)

**Figure 5: Bosch Wheel-Acceleration Based ABS Algorithm**

When the ABS is triggered it enters the first phase of the controller where the brake pressure increases until the peripheral wheel acceleration crosses the threshold (-\(a\)). The controller then switches to holding the brake pressure (Phase 2), to ensure that the tire friction has become fully saturated. Once the slip switching threshold (\(\lambda_1\)) has been reached, the controller will reduce the brake pressure (Phase 3) until the wheel peripheral acceleration exceeds the threshold (-\(a\)). Phase 4 represents a pressure holding phase where the wheel begins to accelerate again as the ring slip enters the stable region of the \(\mu\)-slip curve. Phases 5 through 7 then represent various stages of pressure holding and pressure increases in order to approach the maximum friction coefficient. Once the peripheral wheel acceleration again crosses the threshold (-\(a\)) then the ring slip is assumed to be in
the unstable region. The controller then immediately returns to Phase 3, where the brake pressure is decreased, and the cycle is repeated. Once the estimated vehicle velocity has fallen below a set minimum speed then the controller is deactivated and the brake pressure is allowed to increase, up to the master cylinder pressure, until the vehicle reaches a complete stop.

**Construction of Acceleration Signals (Filtering)**

Since the controller acts upon wheel acceleration thresholds, it is instructive to analyze the open-loop acceleration responses for a tire (Tire 1) following a step increase in valve voltage, as shown in Figure 6. It can be seen that the unfiltered wheel acceleration exhibits large magnitude oscillations before it begins to converge on a specific acceleration. The unfiltered ring acceleration shows oscillations that are smaller, but similar.

![Figure 6: Step-Response of Unfiltered & Filtered Tangential Accelerations for Open-Loop Hub/Tire Model](image)

Figure 6 also shows the open-loop response for Tire 1, with a natural frequency of 48Hz, under different filter settings. For the investigations in this work, the filter type was chosen to be 4\textsuperscript{th}-order Butterworth filter due to its good balance between a reasonable roll-off of 80dB / decade and minimal added phase lag. It can be seen that the 80Hz filter will do little to affect the acceleration signal. However, the 15Hz filter, which will roll off
to -20dB at approximately 27Hz, is decent at filtering out the tire/wheel dynamics. In fact, it is interesting to note that with the 15Hz filter the signal is similar to the unfiltered ring accelerations, but with fewer oscillations, as it is filtering out the torsional dynamics attributed to the sidewall (and the high frequency tread dynamics). The controller is designed to act upon these gradual filtered acceleration changes so that there is a smooth flow between the controller phases.

**Sensitivity Studies**

In the following sub-sections, the interaction between the ABS controller and the tire torsional dynamics will be studied considering changes in the ABS controller filter cutoff frequency, the sidewall damping and stiffness coefficients, and the ring and wheel inertias.

*Effect of Filter Cutoff Frequency*

To analyze the effect of the filter cutoff frequency, the nominal ABS control thresholds are first determined (tuned) for a nominal filter cutoff frequency, and then the filter cutoff frequency is varied while the controller thresholds are held constant. The selected thresholds are those that maintain a sequential flow of the control phases (i.e. the controller should not jump from phase 3 to phase 6, etc…) and minimize both the stopping distance and the control activity. The filter cutoff frequency is varied for a range of tire parameters and the stopping distance and the control activity are analyzed.

Figure 7 shows the sensitivity results where the controller was nominally designed for Tire 1 ($\omega_n = 47.6$ Hz) with a 15Hz filter cutoff frequency, as represented by the black dot. The filter cutoff frequency was then varied between 1.5Hz and 120Hz. The figure also presents a constant ideal stopping distance line that is based on a peak friction coefficient of approximately 0.65, as shown in Figure 4. As expected, the stopping distance and control activity are minimized at the nominal cutoff frequency of 15Hz. It is interesting to note that if the filter cutoff frequency is lowered, below approximately 15 Hz, the stopping distance dramatically increases while the control activity remains small. This can be attributed to a low cutoff frequency filter removing most of the tire-wheel dynamics from the system and resulting in an ABS system that responds too slowly to increases in the
wheel slip. On the other hand, if the cutoff frequency of the filter is increased significantly above the nominal frequency then the control activity begins to increase. This is explained by noting that at these settings, the tire-wheel torsional dynamics have not been sufficiently filtered. This causes the controller to become more active as the most extreme thresholds (\(+A\) and \(-a\)) are easily crossed causing the controller to quickly switch between pressure-increase and pressure-decrease states.

![Figure 7: Cutoff Filter Frequency Sensitivity Study for Tire 1 Parameters](image1)

![Figure 8: Cutoff Filter Frequency Sensitivity Study for Tire 2 Parameters](image2)

Figure 8 shows the results for the stiffer Tire 2 (\(\omega_n = 76.1 \text{ Hz}\)) with a nominal filter cutoff frequency of 25Hz. It can be seen that the control activity will remain relatively small until the filter cutoff frequency reaches approximately 40Hz at which point it begins to increase. This is due to the fact that the torsional stiffness, and thus the natural frequency, is much higher for Tire 2 than for the Tire 1. As such, the filter cutoff frequency can be set at a higher frequency without attenuating much of the tire/wheel sidewall dynamics. If the filter cutoff frequency is lowered below the nominal frequency, then the stopping distance begins to increase because the controller is no longer optimized. However, once the filter cutoff frequency falls below approximately 15Hz then, similar to the Tire 1 case, the stopping distance begins to dramatically increase since most of the tire-wheel dynamics have been removed.
The above analyses suggest that even if the torsional stiffness of the tire is varied, the controller will continue to perform well as long as the cutoff filter frequency satisfies two conditions:

- The filter cutoff frequency must remain above some minimum frequency, in this case approximately 15Hz, for the controller to achieve good performance (stopping distance) with minimal activity.
- The filter cutoff frequency must be set low enough that it will effectively remove the tire/wheel sidewall dynamics from the wheel accelerations used by the controller.

Therefore, when designing the controller and filter parameters, it makes sense to set the filter cutoff frequency close to the lower limit in order to accommodate the largest range of tire torsional stiffnesses, as will be demonstrated further in the following subsection.

**Sensitivity to Sidewall Torsional Stiffness and Damping**

Next we consider the sensitivity of the controller to changes in the sidewall torsional stiffness $K_T$ and the torsional damping coefficient $C_T$. In each case, the controller and filter parameters have been designed for a nominal tire/wheel set. Then, the tire torsional stiffness and damping coefficients are varied while the controller and filter parameters are held constant. Figure 9 shows the sensitivity results where the controller thresholds are optimized for Tire 2, using a 30Hz cutoff filter frequency. The figure shows that as long as the torsional stiffness remains above a certain value, the controller has good performance. If the torsional stiffness is converted into the free-free natural frequency of the system (using Equation 8 and assuming constant inertias), it is found that the stopping distance and control activity begin to increase as the natural frequency of the system approaches the cutoff filter frequency of 30Hz. It can also be seen that the only time there is any significant sensitivity to the damping coefficient is when the controller is already performing poorly due to small values of the torsional stiffness. However, when the torsional stiffness is sufficiently high, the dynamics that are introduced from an over or under-damped system are filtered out of the data and thus will have no effect on the controller. As such, this sensitivity will only be noticed when the system is already not effectively filtering out the natural frequency dynamics due to sidewall flexibility, and thus remains only a secondary concern relative to the sidewall stiffness.
Sensitivity to Wheel and Ring Inertia

It has been shown that the relative value of free-free natural frequency with respect to the filter cutoff frequency has a strong effect on the achievable stopping distance and control activity. It is therefore important to look at all of the factors that affect the natural frequency, including the inertia values of the wheel and the tire ring. Here, this is completed by first tuning the controller to the nominal Tire 2 parameters listed in Table 1, for selected filter cutoff frequencies. The inertias are then varied and the corresponding stopping distance and control activity are recorded. These results are presented in the following plots (Figure 10-Figure 11), where the inertia values are represented as percentage differences from the nominal. Note that the green point represents the nominal inertia values (i.e. $J_r = 100\%$ and $J_w = 100\%$).

Figure 9: $K_t/C_t$ Sensitivity Study for Tire 2 Parameters
As can be seen, at very high values of inertia (increases) there is an increase in control activity and a trend towards an increase in stopping distance. This is due to the fact that at high values of inertia the natural frequency of the system is significantly decreased and begins to approach the filter cutoff frequency. As has already been argued in the previous sections, this response is to be expected. However, it is interesting to note that at small values of ring inertia both the stopping distance and control activity begin to increase again. This is due to the fact that when the tread begins to slip on the ground the ring has no inertia to slow down the transition to full lockup (sliding block).
Conclusions

This work set out to identify and explain influences of ABS controller settings on achievable performance (in stopping distance) as the tire torsional characteristics are changed. It has been identified that filter cutoff frequencies embedded in acceleration-based ABS control strategies play a significant role in influencing the interaction between ABS control activity and tire torsional dynamics.

To summarize these findings:

- The filter cutoff frequency must remain above a certain minimum limit (e.g. 15 Hz for this ABS control algorithm) and yet be set sufficiently low enough to filter out the dynamics from the dominate torsional mode.
- The larger the difference between the filter cutoff frequency and the open-loop free-free natural frequency of the tire-wheel system, the more robust the system becomes to variations in inertia and sidewall stiffness.
- At low values of ring inertia ($J_r$) the controller is unable to control the system and the stopping distance increases sharply.

Accounting for all the potential variations (OEM and aftermarket) in the wheel and tire parameters is vital to ensuring sufficient robustness of the ABS control system in achieving short stopping distances. This can be accomplished by designing the controller around the smallest possible filter cutoff frequency, which will allow for the largest envelope of good performance for various wheel and tire parameters.
References


List of Tables
Table 1: Various Tire Parameters

List of Figure Captions

Figure 1: Hub/Tire Model
Figure 2: Schematic for the LuGre Friction Model
Figure 3: Schematic of Brake Hydraulics
Figure 4: Open-Loop Response with Torsional Dynamics
Figure 5: Bosch Wheel-Acceleration Based ABS Algorithm
Figure 6: Step-Response of Unfiltered & Filtered Tangential Accelerations for Open-Loop Hub/Tire Model
Figure 7: Cutoff Filter Frequency Sensitivity Study for Tire 1 Parameters
Figure 8: Cutoff Filter Frequency Sensitivity Study for Tire 2 Parameters
Figure 9: Kt/Ct Sensitivity Study for Tire 2 Parameters
Figure 10: Jr/Jw Sensitivity Study for Tire 2 -- Tuned for Filter Cutoff Frequency = 30Hz
Figure 11: Jr/Jw Sensitivity Study for Tire 2 -- Tuned for Filter Cutoff Frequency = 35Hz